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Second main theorem and unicity theorem for meromorphic mappings sharing moving hypersurfaces regardless of multiplicity

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Abstract

The purpose of this paper is twofold. The first is to establish a new second main theorem for meromorphic mappings of $\mathbf{C}^m$ into $\mathbf{P}^n(\mathbf{C})$ intersecting moving hypersurfaces with truncated counting functions, where the mappings may be algebraically degenerate. The second is to prove a uniqueness theorem for these mappings which share few moving hypersurfaces without counting multiplicity.

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1. Introduction

In 1926, R. Nevanlinna showed that for two nonconstant meromorphic functions f and g on the complex plane $\mathbf{C}$, if they have the same inverse images for five distinct values then $f = g$, and that g is a special type of linear fractional transformation of f if they have the same inverse images counted with multiplicities for four distinct values [6]. Later on, by applying second main theorems for fixed and moving hyperplanes, many authors have generalized the results of Nevanlinna to the case where meromorphic mappings share fixed or moving hyperplanes without counting multiplicities.

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In 2004, Min Ru [7] showed a second main theorem for algebraically nondegenerate meromorphic mappings and a family of hypersurfaces in weakly general position. With the same assumptions, T.T.H. An and H.T. Phuong [1] improved the result of Min Ru by giving an explicit truncation level for counting functions. Applying the result of An–Phuong, Dulock and Min Ru [2] proved a uniqueness theorem for meromorphic mappings sharing a family of hypersurfaces in weakly general position.

Recently, in [3] Dethloff and Tan generalized and improved the second main theorems of Min Ru and An–Phuong to the case of moving hypersurfaces. They proved the following theorem.

Theorem A. *(See Dethloff–Tan [3].) Let f be a nonconstant meromorphic map of $\mathbf{C}^m$ into $\mathbf{P}^n(\mathbf{C})$. Let $\{Q_i\}_{i=1}^q$ be an admissible set of slow (with respect to f) moving hypersurfaces with $\deg Qj = dj$ $(1 \leqslant i \leqslant q)$. Assume that f is algebraically nondegenerate over $\tilde{\mathcal{K}}_{\{Q_i\}_{i=1}^q}$. Then for any $\epsilon > 0$ there exist positive integers L_j $(j = 1, \ldots, q)$, depending only on n, ϵ and d_j $(j = 1, \ldots, q)$ in an explicit way such that*

$$\|(q-n-1-\epsilon)T_f(r) \leqslant \sum_{i=1}^q \frac{1}{d_i} N_{Q_i(f)}^{[L_j]}(r) + o\big(T_f(r)\big).$$

Here, the truncation level L_j is estimated by

$$L_j \leqslant \frac{d_j \cdot \binom{n+M}{n} t_{p_0+1} - d_j}{d} + 1,$$

where d is the least common multiple of the d_j's, $d = lcm(d_1, \ldots, d_q)$, and

$$M = d \cdot \big[2(n+1)\big(2^n-1\big)(nd+1)\epsilon^{-1} + n + 1\big],$$

$$p_0 = \left[\frac{(\binom{n+M}{n}^2 \cdot \binom{q}{n} - 1) \cdot \log(\binom{n+M}{n}^2 \cdot \binom{q}{n})}{\log(1 + \frac{\epsilon}{2\binom{n+M}{n}N})} + 1\right]^2,$$

and

$$t_{p_0+1} < \left(\binom{n+M}{n}^2 \cdot \binom{q}{n} + p_0\right)^{(\binom{n+M}{n}^2 \cdot \binom{q}{n} - 1)},$$

where $[x] = \max\{k \in \mathbf{Z};\ k \leqslant x\}$ for a real number x.

Applying this theorem, Dethloff and Tan proved a uniqueness theorem for meromorphic mappings sharing slow moving hypersurfaces [4, Theorem 3.1]. Unfortunately, since truncation levels given in Theorem A actually are very weak, the number of moving hypersurfaces needed in the uniqueness theorem of Dethloff–Tan is too big. Also their proof is very complicate.

We also would like to note that, in all mentioned results of Min Ru, An–Phuong and Dethloff–Tan the meromorphic mappings are assumed to be algebraically nondegenerate, and this condition plays an essential role in their proofs.

In this paper, we will give a new second main theorem for meromorphic mappings, which may be algebraically nondegenerate, and slow moving hypersurfaces with better truncation levels for counting functions. Namely, we prove the following.

Theorem 1.1. *Let f be a meromorphic mapping of $\mathbf{C}^m$ into $\mathbf{P}^n(\mathbf{C})$. Let $\mathcal{A} = \{Q_i\}_{i=1}^q$ be a set of slow (with respect to f) moving hypersurfaces of $\mathbf{P}^n(\mathbf{C})$ in general position for the Veronese* *imbedding, where* $\deg Q_i = d_i$ $(1 \leqslant i \leqslant q)$, $d = lcm(d_1, \ldots, d_q)$ *and* $N = \binom{n+d}{n} - 1$. *Assume that* $Q_i(f) \not\equiv 0$ $(1 \leqslant i \leqslant q)$. *Then the following assertions hold*:

a) *If* $q \geqslant 2N+1$ *then*

$$\left\| \frac{q}{2N+1} T_f(r) \leqslant \sum_{\substack{i=1 \\ Q_i(f)\not\equiv 0}}^{q} \frac{1}{d_i} N^{[N]}_{Q_i(f)}(r) + o\big(T_f(r)\big).\right.$$

b) *In addition to the assumptions, we assume further that* $\{Q_i\}_{i=1}^q$ *is in weakly general position. If* $q \geqslant N+n+1$ *then*

$$\left\| \frac{q}{N+n+1} T_f(r) \leqslant \sum_{\substack{i=1 \\ Q_i(f)\not\equiv 0}}^{q} \frac{1}{d_i} N^{[N]}_{Q_i(f)}(r) + o\big(T_f(r)\big).\right.$$

In the case of moving hyperplanes, i.e., $d_i = 1$ $(1 \leqslant i \leqslant q)$, then $N = n$ and Theorem 1.1 gives us the second main theorem for meromorphic mappings and moving hyperplanes with truncated counting functions (see Theorem 2.4 below).

As an application, we prove a uniqueness theorem for meromorphic mappings sharing slow moving hypersurfaces without counting multiplicity as follows.

Theorem 1.2. *Let f and g be meromorphic mappings of* $\mathbf{C}^m$ *into* $\mathbf{P}^n(\mathbf{C})$. *Let* $\{Q_i\}_{i=1}^q$ *be set of slow* (*with respect to f and g*) *moving hypersurfaces in* $\mathbf{P}^n(\mathbf{C})$ *in general position for the Veronese imbedding. Put* $d_i = \deg Q_i$ $(i = 1, \ldots, q)$, $d = lcm(d_1, \ldots, d_q)$ *and* $N = \binom{n+d}{n} - 1$. *Assume that* $Q_i(f) \not\equiv 0$ $(1 \leqslant i \leqslant q)$ *and*

(i) $\dim(\mathrm{Zero}\, Q_i(f) \cap \mathrm{Zero}\, Q_i(f)) \leqslant m-2$ *for every* $1 \leqslant i < j \leqslant q$,

(ii) $f = g$ *on* $\bigcup_{i=1}^q (\mathrm{Zero}\, Q_i(f) \cup \mathrm{Zero}\, Q_i(g))$.

Then the following assertions hold:

a) *If* $q > \frac{2N(2N+1)}{d}$ *then* $f = g$.

b) *In addition to the assumptions, we assume further that* $\{Q_i\}_{i=1}^q$ *is in weakly general position. If* $q > \frac{2N(N+n+1)}{d}$ *then* $f = g$.

We note that the numbers of hypersurfaces in our results are smaller than that in the results of Min Ru–Dulock and Dethloff–Tan. We also simplify their proofs by introducing some new techniques.

2. Basic notions and auxiliary results from Nevanlinna theory

2.1. We set $\|z\| = (|z_1|^2 + \cdots + |z_m|^2)^{1/2}$ for $z = (z_1, \ldots, z_m) \in \mathbf{C}^m$ and define

$$B(r) := \big\{z \in \mathbf{C}^m\colon \|z\| < r\big\}, \qquad S(r) := \big\{z \in \mathbf{C}^m\colon \|z\| = r\big\} \quad (0 < r < \infty).$$

Define

$$v_{m-1}(z) := \big(dd^c\|z\|^2\big)^{m-1} \quad \text{and}$$
$$\sigma_m(z) := d^c \log\|z\|^2 \wedge \big(dd^c \log\|z\|^2\big)^{m-1} \quad \text{on } \mathbf{C}^m \setminus \{0\}.$$

For a divisor ν on $\mathbf{C}^m$ and for a positive integer M or $M = \infty$, we define the counting function of ν by

$$\nu^{[M]}(z)=\min\{M,\nu(z)\},\qquad n(t)=\begin{cases}\int_{|\nu|\cap B(t)}\nu(z)v_{m-1} & \text{if } m\geqslant 2,\\ \sum_{|z|\leqslant t}\nu(z) & \text{if } m=1.\end{cases}$$

Similarly, we define $n^{[M]}(t)$.

Define

$$N(r,\nu)=\int_1^r \frac{n(t)}{t^{2m-1}}\,dt\quad (1<r<\infty).$$

Similarly, we define $N(r,\nu^{[M]})$ and denote it by $N^{[M]}(r,\nu)$.

Let $\varphi:\mathbf{C}^m\to\mathbf{C}$ be a meromorphic function. Denote by ν^0_φ (respectively ν^∞_φ) the zero divisor (respectively pole divisor) of φ. Define

$$N_\varphi(r)=N\big(r,\nu^0_\varphi\big),\qquad N^{[M]}_\varphi(r)=N^{[M]}\big(r,\nu^0_\varphi\big).$$

For brevity we will omit the character $^{[M]}$ if $M=\infty$.

2.2. Let $f:\mathbf{C}^m\to\mathbf{P}^n(\mathbf{C})$ be a meromorphic mapping. For arbitrarily fixed homogeneous coordinates $(w_0:\cdots:w_n)$ on $\mathbf{P}^n(\mathbf{C})$, we take a reduced representation $f=(f_0:\cdots:f_n)$, which means that each f_i is a holomorphic function on $\mathbf{C}^m$ and $f(z)=(f_0(z):\cdots:f_n(z))$ outside the analytic subset $\{f_0=\cdots=f_n=0\}$ of codimension $\geqslant 2$. Set $\|f\|=(|f_0|^2+\cdots+|f_n|^2)^{1/2}$.

The characteristic function of f is defined by

$$T_f(r)=\int_{S(r)}\log\|f\|\sigma_m-\int_{S(1)}\log\|f\|\sigma_m.$$

2.3. Let φ be a nonzero meromorphic function on $\mathbf{C}^m$, which are occasionally regarded as a meromorphic map into $\mathbf{P}^1(\mathbf{C})$. The proximity function of φ is defined by

$$m(r,\varphi):=\int_{S(r)}\log\max\big(|\varphi|,1\big)\sigma_m.$$

The Nevanlinna's characteristic function of φ is define as follows

$$T(r,\varphi):=N_{\frac{1}{\varphi}}(r)+m(r,\varphi).$$

Then

$$T_\varphi(r)=T(r,\varphi)+O(1).$$

The function φ is said to be small (with respect to f) if $\|T_\varphi(r)=o(T_f(r))$. Here, by the notation "$\|P$" we mean the assertion P holds for all $r\in[0,\infty)$ excluding a Borel subset E of the interval $[0,\infty)$ with $\int_E dr<\infty$.

We denote by $\mathcal{M}$ (respectively $\mathcal{K}_f$) the field of all meromorphic functions (respectively small meromorphic functions) on $\mathbf{C}^m$.

2.4. Denote by $\mathcal{H}_{\mathbf{C}^m}$ the ring of all holomorphic functions on $\mathbf{C}^m$. Let Q be a homogeneous polynomial in $\mathcal{H}_{\mathbf{C}^m}[x_0,\dots,x_n]$ of degree $d\geqslant 1$. Denote by $Q(z)$ the homogeneous polynomial over $\mathbf{C}$ obtained by substituting a specific point $z\in\mathbf{C}^m$ into the coefficients of Q. We also call a moving hypersurface in $\mathbf{P}^n(\mathbf{C})$ each homogeneous polynomial $Q\in\mathcal{H}_{\mathbf{C}^m}[x_0,\dots,x_n]$ such that the common zero set of all coefficients of Q has codimension at least two.

Let Q be a moving hypersurface in $\mathbf{P}^n(\mathbf{C})$ of degree $d \geqslant 1$ given by

$$Q(z) = \sum_{I \in \mathcal{I}_d} a_I \omega^I,$$

where $\mathcal{I}_d = \{(i_0, \ldots, i_n) \in \mathbf{N}_0^{n+1};\ i_0 + \cdots + i_n = d\}$, $a_I \in \mathcal{H}_{\mathbf{C}^m}$ and $\omega^I = \omega_0^{i_0} \cdots \omega_n^{i_n}$. We consider the meromorphic mapping $Q' : \mathbf{C}^m \to \mathbf{P}^N(\mathbf{C})$, where $N = \binom{n+d}{n} - 1$, given by

$$Q'(z) = \big(a_{I_0}(z) : \cdots : a_{I_N}(z)\big) \quad \big(\mathcal{I}_d = \{I_0, \ldots, I_N\}\big).$$

The moving hypersurface Q is said to be "slow" (with respect to f) if $\| T_{Q'}(r) = o(T_f(r))$. This is equivalent to $\| T_{\frac{a_{I_i}}{a_{I_j}}}(r) = o(T_f(r))$ for every $a_{I_j} \not\equiv 0$. We denote by $v_Q = (a_{I_0}, \ldots, a_{I_N}) \in \mathcal{H}_{\mathbf{C}^m}^{N+1}$ the vector associated with Q.

Let $\{Q_i\}_{i=1}^q$ be a family of moving hypersurfaces in $\mathbf{P}^n(\mathbf{C})$, $\deg Q_i = d_i$. Assume that

$$Q_i = \sum_{I \in \mathcal{I}_{d_i}} a_{iI} \omega^I.$$

We denote by $\tilde{\mathcal{K}}_{\{Q_i\}_{i=1}^q}$ the smallest subfield of $\mathcal{M}$ which contains $\mathbf{C}$ and all $\frac{a_{iI}}{a_{iJ}}$ with $a_{iJ} \not\equiv 0$. We say that $\{Q_j\}_{j=1}^q$ are in weakly general position if there exists $z \in \mathbf{C}^m$ such that for any $1 \leqslant j_0 < \cdots < j_n \leqslant q$ the system of equations

$$\begin{cases} Q_{j_i}(z)(w_0, \ldots, w_n) = 0, \\ 0 \leqslant i \leqslant n \end{cases}$$

has only the trivial solution $w = (0, \ldots, 0)$ in $\mathbf{C}^{n+1}$.

Setting $d = lcm(d_1, \ldots, d_q)$ and $N = \binom{n+d}{n} - 1$, we say that the family $\{Q_i\}_{i=1}^q$ is in general position for the Veronese imbedding if for any $1 \leqslant i_0 < \cdots < i_n \leqslant q$, the set $\{(v_{Q_{i_0}^{d/d_{i_0}}}, \ldots, v_{Q_{i_n}^{d/d_{i_n}}})\}$ is linearly independent over $\tilde{\mathcal{K}}_{\{Q_i\}_{i=1}^q}$.

2.5. Let f be a nonconstant meromorphic map of $\mathbf{C}^m$ into $\mathbf{P}^n(\mathbf{C})$. Denote by $\mathcal{C}_f$ the set of all nonnegative functions $h : \mathbf{C}^m \setminus A \to [0, +\infty] \subset \overline{\mathbf{R}}$, which are of the form

$$h = \frac{|g_1| + \cdots + |g_l|}{|g_{l+1}| + \cdots + |g_{l+k}|},$$

where $k, l \in \mathbf{N}$, $g_1, \ldots, g_{l+k} \in \mathcal{K}_f \setminus \{0\}$ and $A \subset \mathbf{C}^m$, which may depend on $g_1, \ldots, g_{l+k}$, is an analytic set of codimension at least two. Then, for $h \in \mathcal{C}_f$ we have

$$\int_{S(r)} \log h \sigma_m = o\big(T_f(r)\big).$$

Lemma 2.1. *(See [3, Lemma 2].) Let $\{Q_i\}_{i=0}^n$ be a set of homogeneous polynomials of degree d in $\mathcal{K}_f[x_0, \ldots, x_n]$. Then there exists a function $h_1 \in \mathcal{C}_f$ such that, outside an analytic set of $\mathbf{C}^m$ of codimension at least two,*

$$\max_{i \in \{0, \ldots, n\}} \big|Q_i(f_0, \ldots, f_n)\big| \leqslant h_1 \|f\|^d.$$

If, moreover, this set of homogeneous polynomials is in weakly general position, then there exists a nonzero function $h_2 \in \mathcal{C}_f$ such that, outside an analytic set of $\mathbf{C}^m$ of codimension at least two,

$$h_2 \|f\|^d \leqslant \max_{i \in \{0,\dots,n\}} \left|Q_i(f_0,\dots,f_n)\right|.$$

Lemma 2.2 *(Lemma on logarithmic derivative). (See [8, Lemma 3.11].) Let f be a nonzero meromorphic function on $\mathbf{C}^m$. Then*

$$\left\| m\left(r, \frac{\mathcal{D}^\alpha(f)}{f}\right) = O\left(\log^+ T(r,f)\right) \quad \left(\alpha \in \mathbf{Z}^n_+\right).\right.$$

2.6. Assume that $\mathcal{L}$ is a subset of a vector space V over a field $\mathcal{R}$. We say that the set $\mathcal{L}$ is *minimal* over $\mathcal{R}$ if it is linearly dependent over $\mathcal{R}$ and each proper subset of $\mathcal{L}$ is linearly independent over $\mathcal{R}$.

Repeating the argument in [5, Proposition 4.5], we have the following:

Proposition 2.3. *Let $\Phi_0,\dots,\Phi_k$ be meromorphic functions on $\mathbf{C}^m$ such that $\{\Phi_0,\dots,\Phi_k\}$ are linearly independent over $\mathbf{C}$. Then there exist an admissible set*

$$\left\{\alpha_i = (\alpha_{i1},\dots,\alpha_{im})\right\}_{i=0}^k \subset \mathbf{Z}^m_+$$

with $|\alpha_i| = \sum_{j=1}^m |\alpha_{ij}| \leqslant k$ $(0 \leqslant i \leqslant k)$ such that the following are satisfied:

(i) $\{\mathcal{D}^{\alpha_i}\Phi_0,\dots,\mathcal{D}^{\alpha_i}\Phi_k\}_{i=0}^k$ *is linearly independent over $\mathcal{M}$, i.e.,* $\det(\mathcal{D}^{\alpha_i}\Phi_j) \not\equiv 0$.

(ii) $\det(\mathcal{D}^{\alpha_i}(h\Phi_j)) = h^{k+1} \cdot \det(\mathcal{D}^{\alpha_i}\Phi_j)$ *for any nonzero meromorphic function h on* $\mathbf{C}^m$.

Theorem 2.4. *(See [9, Corollary 1].) Let $f : \mathbf{C}^m \to \mathbf{P}^n(\mathbf{C})$ be a meromorphic mapping. Let $\{a_i\}_{i=1}^q$ $(q \geqslant 2n+1)$ be a set of q small (with respect to f) meromorphic mappings of $\mathbf{C}^m$ into $\mathbf{P}^n(\mathbf{C})$ in general position. Then*

$$\left\| \frac{q}{2n+1} \cdot T_f(r) \leqslant \sum_{\substack{i=1 \\ (f,a_i)\not\equiv 0}}^q N^{[n]}_{(f,a_i)}(r) + o\left(T_f(r)\right).\right.$$

3. Second main theorem for moving hypersurfaces

In order to prove Theorem 1.1 we need some following lemmas.

Lemma 3.1. *Let f be as in Theorem* 1.1. *Let $\{Q_i\}_{i=0}^{N+n}$ be a set of homogeneous polynomials in $\mathcal{K}_f[x_0,\dots,x_n]$ in general position for the Veronese imbedding and in weakly general position in $\mathbf{P}^n(\mathbf{C})$ of common degree d, where $N = \binom{n+d}{n} - 1$. Assume that $Q_0(f) \not\equiv 0$. Then there exist subsets $I_1,\dots,I_k$ of $\{Q_i(f)\}_{i=1}^{N+n}$ such that the following are satisfied*:

(i) $\{0\} \cup I_i$ *is minimal and I_i is linearly independent over $\mathcal{R}_f$ $(1 \leqslant i \leqslant k)$.*

(ii) $\sharp(\bigcup_{i=1}^k I_i) \geqslant n$.

Proof. Denote by V^d_f the vector space of all homogeneous polynomial of degree d in $\mathcal{K}_f[x_0,\dots,x_n]$ over the field $\mathcal{K}_f$. It is seen that $\dim V^d_f = \binom{n+d}{n} = N+1$. We set $V^d_f(f) = \{Q(f);\ Q \in V^d_f\}$, which is a vector space over $\mathcal{K}_f$.

We are going to construct the subset I_i as follows.

We set $A_1 = \{Q_i(f);\ 1 \leqslant i \leqslant N+n\}$. Since $\sharp A_1 \geqslant N+1 \geqslant \dim V_f^d(f)$ and $\{Q_i\}_{i=0}^{N+n}$ is assumed to be in general position for the Veronese imbedding, $(A_1)_{\mathcal{K}_f} = V_f^d(f)$. Hence $Q_0(f) \in (A_1)_{\mathcal{K}_f}$. We choose a subset I_1 of A_1 which is the minimal subset of A_1 satisfying $Q_0(f) \in (I_1)_{\mathcal{K}_f}$. By the minimality, the subset I_1 is linearly independent and the set $\{Q_0(f)\} \cup I_1$ is minimal over $\mathcal{K}_f$. If $\sharp I_1 \geqslant n$ then we stop the process.

Otherwise, set $A_2 = A_1 \setminus I_1$. Since $\sharp A_2 \geqslant \sharp A_1 - n \geqslant N+1 \geqslant \dim V_f^d(f)$ and $\{Q_i\}_{i=0}^{N+n}$ is assumed to be in general position for the Veronese imbedding, $(A_2)_{\mathcal{K}_f} = V_f^d(f)$. Similarly as above, we choose a subset I_2 of A_2 such that I_2 is the minimal subset of A_2 satisfying $Q_0(f) \in (I_1)_{\mathcal{K}_f}$. It is clear that the subset I_2 is linearly independent and the subset $\{Q_0(f)\} \cup I_2$ is minimal over $\mathcal{K}_f$. If $\sharp(I_1 \cup I_2) \geqslant n$ then we stop the process.

Otherwise, by repeating the above argument, we have a subset I_3 of $A_3 = A_1 \setminus (I_1 \cup I_2)$.

Continuing this process, then there exist the subsets $I_1, \ldots, I_k$ such that: I_i is a subset of $A_1 \setminus \bigcup_{j=1}^{i-1} I_j$, I_i is linearly independent and $\{Q_0(f)\} \cup I_i$ is minimal over $\mathcal{K}_f$ $(2 \leqslant j \leqslant k)$, $\sharp(\bigcup_{i=1}^{k} I_k) \geqslant n$.

We finish the proof of the lemma. $\square$

Lemma 3.2. *Let f be as in Theorem* 1.1. *Let $\{Q_i\}_{i=0}^{N+n}$ be a set of homogeneous polynomials in $\mathcal{K}_f[x_0, \ldots, x_n]$ of common degree d in general position for the Veronese imbedding and in weakly general position, where $N = \binom{n+d}{n} - 1$. Then we have*

$$\|T_f(r) \leqslant \sum_{\substack{i=0\\ Q_i(f)\not\equiv 0}}^{N+n} \frac{1}{d} N_{Q_i(f)}^{[N]}(r) + o\big(T_f(r)\big).$$

Proof. Without loss of generality, we assume that $Q_0(f) \not\equiv 0$. By Lemma 3.1, we may assume that there exist the subsets $I_i = \{Q_{t_i+1}(f), \ldots, Q_{t_{i+1}}(f)\}$ $(1 \leqslant i \leqslant k)$, where $t_1 = 0$, which satisfy the assertions of Lemma 3.1. By the minimality over $\mathcal{K}_f$ of the sets $\{Q_0(f)\} \cup I_i$, it follows that $Q_i(f) \not\equiv 0$ $(1 \leqslant i \leqslant t_{k+1})$ and there exist functions $c_i \in \mathcal{K}_f \setminus \{0\}$ $(1 \leqslant i \leqslant t_{k+1})$ such that

$$Q_0(f) + \sum_{j=1}^{t_{i+1}-t_i} c_{t_i+j} Q_{t_i+j}(f) = 0 \quad (1 \leqslant i \leqslant k).$$

We set $c_{i0} = 1$ for all $i = 1, \ldots, k$.

Consider $i \geqslant 1$. We set $c_{ij} = c_j$ for all $j \in \{t_i+1, \ldots, t_{i+1}\}$ and $c_{ij} = 0$ for all $0 < j \leqslant t_i$ or $j > t_{i+1}$. Then we have

$$c_{i0} Q_0(f) + \sum_{j=t_i+1}^{t_{i+1}} c_{ij} Q_j(f) = 0.$$

Since $\{c_{ij} Q_j(f)\}_{t_i+1 \leqslant j \leqslant t_{i+1}}$ is linearly independent over $\mathcal{K}_f$, there exists an admissible set $\{\alpha_{t_i+1}, \ldots, \alpha_{t_{i+1}}\} \subset \mathbf{Z}_+^m$ $(|\alpha_s| \leqslant t_{i+1} - t_i - 1 \leqslant N)$ such that

$$\begin{aligned} A_i &= \det\big(\mathcal{D}^{\alpha_s}\big(c_{ij} Q_j(f)\big)\big)_{t_i+1 \leqslant s, j \leqslant t_{i+1}} \\ &= \big(Q_0(f)\big)^{t_{i+1}-t_i} \cdot \det\left(\mathcal{D}^{\alpha_s}\left(\frac{c_{ij} Q_j(f)}{Q_0(f)}\right)\right)_{t_i+1 \leqslant s, j \leqslant t_{i+1}} \end{aligned}$$

$$= \big(Q_0(f)\big)^{t_{i+1}-t_i} \cdot \tilde{A}_i \not\equiv 0.$$

For each s $(1 \leqslant s \leqslant t_{k+1})$, taking i so that $t_i + 1 \leqslant s \leqslant t_{i+1}$, we set $c_{i_s j} = c_{ij}$. Consider an $t_{k+1} \times (t_{k+1}+1)$ minor matrixes $\mathcal{T}$ and $\tilde{\mathcal{T}}$ given by

$$\mathcal{T} = \big[\mathcal{D}^{\alpha_s}\big(c_{i_s j} Q_j(f)\big);\ 0 \leqslant j \leqslant t_{k+1}\big]_{1\leqslant s\leqslant t_{k+1}},$$
$$\tilde{\mathcal{T}} = \left[\mathcal{D}^{\alpha_s}\left(\frac{c_{i_s j} Q_j(f)}{Q_0(f)}\right);\ 0 \leqslant j \leqslant t_{k+1}\right]_{1\leqslant s\leqslant t_{k+1}}.$$

Denote by D_i (respectively $\tilde{D}_i$) the determinant of the matrix obtained by deleting the $(i+1)$-th column of the minor matrix $\mathcal{T}$ (respectively $\tilde{\mathcal{T}}$). It is clear that the sum of each row of $\mathcal{T}$ (respectively $\tilde{\mathcal{T}}$) is zero, then we have

$$D_i = (-1)^i D_0 = (-1)^i \prod_{i=1}^{k} A_i = (-1)^i \big(Q_0(f)\big)^{t_{k+1}} \prod_{i=1}^{k} \tilde{A}_i$$
$$= (-1)^i \big(Q_0(f)\big)^{t_{k+1}} \tilde{D}_0 = \big(Q_0(f)\big)^{t_{k+1}} \tilde{D}_i.$$

Since $\sharp(\{Q_0(f)\} \cup \bigcup_{i=1}^{k} I_i) \geqslant n+1$ and $Q_0, \dots, Q_{t_{k+1}}$ are in weakly general position, by Lemma 2.1 there exists a function $\Psi \in \mathcal{C}_f$ such that

$$\|f(z)\|^d \leqslant \Psi(z) \cdot \max_{0\leqslant i\leqslant t_{k+1}} \big(\big|Q_i(f)(z)\big|\big) \quad \big(z \in \mathbf{C}^m\big).$$

Fix $z_0 \in \mathbf{C}^m$. Take i $(0 \leqslant i \leqslant t_{k+1})$ such that $|Q_i(f)(z_0)| = \max_{0\leqslant j\leqslant t_{k+1}} |Q_j(f)(z_0)|$. Then

$$\frac{|D_0(z_0)| \cdot \|f(z_0)\|^d}{\prod_{j=0}^{t_{k+1}} |Q_j(f)(z_0)|} = \frac{|D_i(z_0)|}{\prod_{\substack{j=0\\ j\neq i}}^{t_{k+1}} |Q_j(f)(z_0)|} \cdot \left(\frac{\|f(z_0)\|^d}{|Q_i(f)(z_0)|}\right)$$
$$\leqslant \Psi(z_0) \cdot \frac{|D_i(z_0)|}{\prod_{\substack{j=0\\ j\neq i}}^{t_{k+1}} |Q_j(f)(z_0)|}.$$

This implies that

$$\log \frac{|D_0(z_0)|.\|f(z_0)\|^d}{\prod_{j=0}^{t_{k+1}} |Q_j(f)(z_0)|} \leqslant \log^+\left(\Psi(z_0) \cdot \left(\frac{|D_i(z_0)|}{\prod_{j=0, j\neq i}^{t_{k+1}} |Q_j(f)(z_0)|}\right)\right)$$
$$\leqslant \log^+\left(\frac{|D_i(z_0)|}{\prod_{j=0, j\neq i}^{t_{k+1}} |Q_j(f)(z_0)|}\right) + \log^+ \Psi(z_0).$$

Thus, for each $z \in \mathbf{C}^m$, we have

$$\log \frac{|D_0(z)|.\|f(z)\|^d}{\prod_{i=0}^{t_{k+1}} |Q_i(f)(z)|} \leqslant \sum_{i=0}^{t_{k+1}} \log^+\left(\frac{|D_i(z)|}{\prod_{j=0, j\neq i}^{t_{k+1}} |Q_j(f)(z)|}\right) + \log^+ \Psi(z)$$
$$= \sum_{i=0}^{t_{k+1}} \log^+\left(\frac{|\tilde{D}_i(z)|}{\prod_{j=0, j\neq i}^{t_{k+1}} |\frac{Q_j(f)(z)}{Q_0(f)(z)}|}\right) + \log^+ \Psi(z). \tag{3.1}$$

Note that $\frac{\tilde{D}_i}{\prod_{j=0, j\neq i}^{t_{k+1}} \frac{Q_j(f)}{Q_0(f)}} = \det[\frac{\mathcal{D}^{\alpha_s}(\frac{c_{i_s j} Q_j(f)}{Q_0(f)})}{\frac{c_{i_s j} Q_j(f)}{Q_0(f)}};\ 0 \leqslant j \leqslant t_{k+1},\ j \neq i]_{1\leqslant s\leqslant t_{k+1}}$.

(The determinant is counted after deleting the i-th column in the above matrix.)

By the lemma on logarithmic derivative, for each i and $c \in \mathcal{K}_f$ we have

$$\begin{aligned}\Big\| m\left(r, \frac{\mathcal{D}^\alpha(\frac{cQ_j(f)}{Q_0(f)})}{\frac{Q_j(f)}{Q_0(f)}}\right) &\leqslant m\left(r, \frac{\mathcal{D}^\alpha(\frac{cQ_j(f)}{Q_0(f)})}{\frac{cQ_j(f)}{Q_0(f)}}\right) + m(r,c) \\ &\leqslant O\big(\log^+ T_{\frac{cQ_j(f)}{Q_0(f)}}(r)\big) + T_c(r) = o\big(T_f(r)\big).\end{aligned}$$

Therefore, we have

$$\Big\| m\left(r, \frac{\tilde{D}_i}{\prod_{j=0, j\neq i}^{t_{k+1}} \frac{Q_j(f)}{Q_0(f)}}\right) = o\big(T_f(r)\big) \quad (0 \leqslant i \leqslant t_k).$$

Integrating both sides of the inequality (3.1), we get

$$\begin{aligned}&\Big\| \int_{S(r)} \log\|f\|^d \sigma_m + \int_{S(r)} \log\left(\frac{|D_0|}{\prod_{i=0}^{t_{k+1}} |Q_i(f)|}\right)\sigma_m \\ &\leqslant \sum_{i=0}^{t_{k+1}} \int_{S(r)} \log^+\left(\frac{|\tilde{D}_i|}{\prod_{j=0, j\neq i}^{t_{k+1}} |\frac{Q_j(f)}{Q_0(f)}|}\right)\sigma_m + \int_{S(r)} \log^+ \Psi(z)\sigma_m \\ &\leqslant \sum_{i=0}^{t_{k+1}} m\left(r, \frac{\tilde{D}_i}{\prod_{j=0, j\neq i}^{t_{k+1}} \frac{Q_j(f)}{Q_0(f)}}\right) + o\big(T_f(r)\big) = o\big(T_f(r)\big).\end{aligned}$$

By Jensen formula, the above inequality implies that

$$\| dT_f(r) + N_{D_0}(r) - N_{\frac{1}{D_0}}(r) - \sum_{i=0}^{t_{k+1}} N_{Q_i(f)}(r) \leqslant o\big(T_f(r)\big). \tag{3.2}$$

It is easy to see that a pole of D_0 must be pole of some c_{is} or pole of some nonzero coefficients a_{iI} of Q_i and

$$N_{\frac{1}{D_0}}(r) \leqslant O\left(\sum_{i,s} N_{\frac{1}{c_{is}}}(r) + \sum_{a_{iI}\not\equiv 0} N_{\frac{1}{a_{iI}}}(r)\right) = o\big(T_f(r)\big).$$

Therefore, the inequality (3.2) implies that

$$\| dT_f(r) \leqslant \sum_{i=0}^{t_{k+1}} N_{Q_i(f)}(r) - N_{D_0}(r) + o\big(T_f(r)\big). \tag{3.3}$$

Here we note that $D_i = (-1)^i D_0$, then $\nu^0_{D_i} = \nu^0_{D_0}$ for each i ($1 \leqslant i \leqslant t_{k+1}$).

We now assume that z is a zero of some functions $Q_i(f)$. Since $t_{k+1} + 1 \geqslant n+1$ and z cannot be zero of more than n functions $Q_i(f)$, without loss of generality we may assume that z is not zero of $Q_0(f)$. Then

$$\begin{aligned}&\nu^0_{\mathcal{D}^{\alpha_{st_{s-1}+j}}(c_{si}Q_i(f))}(z) \\ &\geqslant \min_{\beta\in\mathbf{Z}^m_+ \text{ with } \alpha_{st_{s-1}+j}-\beta\in\mathbf{Z}^m_+} \big\{\nu^0_{\mathcal{D}^\beta c_{si}\mathcal{D}^{\alpha_{st_{s-1}+j}-\beta} Q_i(f)}(z)\big\}\end{aligned}$$

$$\geqslant \min_{\beta\in\mathbf{Z}_+^m \text{ with } \alpha_{st_{s-1}+j}-\beta\in\mathbf{Z}_+^m}\big\{\max\big\{0,\nu^0_{Q_i(f)}(z)-|\alpha_{st_{s-1}+j}-\beta|\big\}-(\beta+1)\nu^\infty_{c_{si}}(z)\big\}$$
$$\geqslant \max\big\{0,\nu^0_{Q_i(f)}(z)-N\big\}-(N+1)\nu^\infty_{c_{si}}(z)$$

for each $1\leqslant i\leqslant t_{k+1}$, $1\leqslant j\leqslant t_s-t_{s-1}$, $1\leqslant s\leqslant k+1$, where $t_0=0$.

Put $I(z)=(N+1)\sum_{s=1}^{k+1}\sum_{i=0}^{t_{k+1}}(t_s-t_{s-1})\nu^\infty_{c_{si}}(z)$. Then

$$\nu_{D_0}(z)\geqslant\sum_{i=0}^{t_{k+1}}\max\big\{0,\nu^0_{Q_i(f)}(z)-N\big\}-I(z). \tag{3.4}$$

We note that if z is not zero of a function $Q_i(f)$ with $i\neq 0$, replacing D_0 by D_i and repeating the same argument we again get the inequality (3.4). Hence (3.4) holds for all $z\in\mathbf{C}^m$. It follows that

$$\sum_{i=0}^{t_{k+1}}\nu^0_{Q_i(f)}(z)-\nu_{D_0}(z)\leqslant\sum_{i=0}^{t_{k+1}}\min\big\{N,\nu^0_{Q_i(f)}(z)\big\}+I(z).$$

Integrating both sides of the above inequality, we get

$$\sum_{i=0}^{t_{k+1}}N_{Q_i(f)}(r)-N_{D_0}(r)\leqslant\sum_{i=0}^{t_{k+1}}N^{[N]}_{Q_i(f)}(r)+o\big(T_f(r)\big).$$

Combining this and (3.3), we get

$$\|T_f(r)\leqslant\sum_{\substack{i=0\\ Q_i(f)\not\equiv 0}}^{N+n}\frac{1}{d}N^{[N]}_{Q_i(f)}(r)+o\big(T_f(r)\big).$$

The lemma is proved. □

Proof of Theorem 1.1. a) We first prove the assertion a) for the case where all Q_i $(i=1,\dots,q)$ have the same degree d.

Fix homogeneous coordinates $(w_0:\cdots:w_n)$ of $\mathbf{P}^n(\mathbf{C})$. We assume that f has a reduced representation $f=(f_0:\cdots:f_n)$ and each Q_i is given by the homogeneous polynomial

$$Q_i=\sum_I a_{iI}\omega_0^{i_0}\cdots\omega_n^{i_n}\quad(1\leqslant i\leqslant q),$$

where $I=(i_0,\dots,i_n)$ with $i_0+\cdots+i_n=d$ and each a_{iI} is holomorphic function on $\mathbf{C}^m$.

We define a meromorphic mapping F of $\mathbf{C}^m$ into $\mathbf{P}^N(\mathbf{C})$ by $F:=(\cdots:F_I:\cdots)$, where $F_I=f_0^{i_0}\cdots f_n^{i_n}$, and define moving hyperplanes H_i of $\mathbf{P}^N(\mathbf{C})$ by

$$H_i=\sum_I a_{iI}W_I\quad(1\leqslant i\leqslant q),$$

where $(\cdots:W_I:\cdots)$ denotes the homogeneous coordinates of $\mathbf{P}^N(\mathbf{C})$.

It is clear that $dT_f(r)=T_F(r)$ and all H_i $(1\leqslant i\leqslant q)$ are slow (with respect to F) moving hyperplanes of $\mathbf{P}^N(\mathbf{C})$ in weakly general position. Applying the second main theorem for F and moving hyperplanes, we have

$$\left\|\frac{q}{2N+1}T_F(r)\leqslant\sum_{i=1}^q N^{[N]}_{H_i(F)}(r)+o\big(T_F(r)\big)=\sum_{i=1}^q N^{[N]}_{Q_i(f)}(r)+o\big(T_F(r)\big).\right.$$

This implies that

$$\left\| \frac{q}{2N+1} T_f(r) \leqslant \sum_{i=1}^{q} \frac{1}{d} N^{[N]}_{Q_i(f)}(r) + o\big(T_f(r)\big).\right.$$

Hence we have the derised inequality in this case.

We now prove the assertion a) for the general case where $\deg Q_i = d_i$. Then, applying the above case for f and the moving hypersurfaces $Q_i^{\frac{d}{d_i}}$ $(i = 1, \ldots, q)$ of common degree d, we have

$$\begin{aligned}\left\| \frac{q}{2N+1} T_f(r)\right. &\leqslant \sum_{j=1}^{q} \frac{1}{d} N^{[N]}_{Q_i^{d/d_i}(f)}(r) + o\big(T_f(r)\big) \\ &\leqslant \sum_{j=1}^{q} \frac{1}{d}\frac{d}{d_i} N^{[N]}_{Q_i(f)}(r) + o\big(T_f(r)\big) \\ &= \sum_{j=1}^{q} \frac{1}{d_i} N^{[N]}_{Q_i(f)}(r) + o\big(T_f(r)\big).\end{aligned}$$

Then, the assertion a) is proved.

b) By repeating the argument as in the proof of the assertion a), it suffices to prove for the case where all Q_i have the same degree. By changing the homogeneous coordinates of $\mathbf{P}^n(\mathbf{C})$ if necessary, we may assume that $a_{iI_1} \not\equiv 0$ for every $i = 1, \ldots, q$. We set $\tilde{Q}_i = \frac{1}{a_{iI_1}} Q_i$. Then $\{\tilde{Q}_i\}_{i=1}^{q}$ is a set of homogeneous polynomials in $\mathcal{K}_f[x_0, \ldots, x_n]$ in general position for Veronese imbedding and in weakly general position.

Consider $(N+n+1)$ polynomials $\tilde{Q}_{i_1}, \ldots, \tilde{Q}_{i_{N+n+1}}$ $(1 \leqslant i_j \leqslant q)$. Applying Lemma 3.2, we have

$$\left\| T_f(r) \leqslant \sum_{j=1}^{N+n+1} \frac{1}{d} N^{[N]}_{\tilde{Q}_{i_j}(f)}(r) + o\big(T_f(r)\big) \leqslant \sum_{\substack{j=1 \\ Q_{i_j}(f)\not\equiv 0}}^{N+n+1} \frac{1}{d} N^{[N]}_{Q_{i_j}(f)}(r) + o\big(T_f(r)\big).\right.$$

Taking summing-up of both sides of this inequality over all combinations $\{i_1, \ldots, i_{N+n+1}\}$ with $1 \leqslant i_1 < \cdots < i_{N+n+1} \leqslant q$, we have

$$\left\| \frac{q}{N+n+1} T_f(r) \leqslant \sum_{\substack{i=1 \\ Q_i(f)\not\equiv 0}}^{q} \frac{1}{d} N^{[N]}_{Q_i(f)}(r) + o\big(T_f(r)\big).\right.$$

The assertion b) is proved. □

4. Uniqueness theorems for meromorphic mappings sharing moving hypersurfaces

Lemma 4.1. *Let f and g be nonconstant meromorphic mappings of $\mathbf{C}^m$ into $\mathbf{P}^n(\mathbf{C})$. Let Q_i $(i = 1, \ldots, q)$ be slow (with respect to f and g) moving hypersurfaces in $\mathbf{P}^n(\mathbf{C})$ of degree d_i in general position for Veronese imbedding. Put $d = lcm(d_1, \ldots, d_q)$ and $N = \binom{n+d}{n} - 1$. Then the following assertions hold:*

(i) *If* $q > \frac{2N(2N+1)}{d}$ *then* $\| T_f(r) = O(T_g(r))$ *and* $\| T_g(r) = O(T_f(r))$.

(ii) *In addition to the assumption, we assume that* $\{Q_i\}_{i=1}^q$ *is in weakly general position. If* $q > \frac{2N(N+n+1)}{d}$ *then* $\| T_f(r) = O(T_g(r))$ *and* $\| T_g(r) = O(T_f(r))$.

Proof. (i) We now prove the assertion (i). It is clear that $q \geqslant N + n + 1$. Then using Theorem 1.1a) for g, we have

$$\begin{aligned}\Big\| \frac{q}{2N+1} T_g(r) &\leqslant \sum_{i=1}^q \frac{1}{d_i} N^{[N]}_{Q_i(g)}(r) + o\big(T_g(r)\big)\\ &\leqslant \sum_{i=1}^q \frac{N}{d_i} N^{[1]}_{Q_i(g)}(r) + o\big(T_g(r)\big)\\ &= \sum_{i=1}^q \frac{N}{d_i} N^{[1]}_{Q_i(f)}(r) + o\big(T_g(r)\big)\\ &\leqslant qN\, T_f(r) + o\big(T_g(r)\big).\end{aligned}$$

Hence $\| T_g(r) = O(T_f(r))$. Similarly, we get $\| T_f(r) = O(T_g(r))$.

(ii) By using Theorem 1.1b) instead of Theorem 1.1a) in the proof of the first assertion, we will get the proof of the second one. □

Proof of Theorem 1.2. We assume that f and g have reduced representations $f = (f_0 : \cdots : f_n)$ and $g = (g_0 : \cdots : g_n)$ respectively. Replacing Q_i by $Q_i^{\frac{d}{d_i}}$ if necessary, without loss of generality, we may assume that $d_i = d$ for every $i = 1, \ldots, q$.

a) By Lemma 4.1(i), we have $\| T_f(r) = O(T_g(r))$ and $\| T_g(r) = O(T_f(r))$. Suppose that $f \not\equiv g$. Then there exist two indices s, t $(0 \leqslant s < t \leqslant n)$ satisfying

$$H := f_s g_t - f_t g_s \not\equiv 0.$$

By the assumption (ii) of the theorem, we have $H = 0$ on $\bigcup_{i=1}^q (\mathrm{Zero}\, Q_i(f) \cup \mathrm{Zero}\, Q_i(g))$. Therefore, we have

$$\nu^0_H \geqslant \sum_{i=1}^q \min\big\{1, \nu^0_{Q_i(f)}\big\}$$

outside an analytic subset of codimension at least two. Then, it follows that

$$N_H(r) \geqslant \sum_{i=1}^q N^{[1]}_{Q_i(f)}(r). \tag{4.1}$$

On the other hand, by the definition of the characteristic function and Jensen formula, we have

$$N_H(r) = \int_{S(r)} \log|f_s g_t - f_t g_s|\sigma_m \leqslant \int_{S(r)} \log\|f\|\sigma_m + \int_{S(r)} \log\|g\|\sigma_m = T_f(r) + T_g(r).$$

Combining this and (4.1), we obtain

$$T_f(r) + T_g(r) \geqslant \sum_{i=1}^q N^{[1]}_{Q_i(f)}(r).$$

Similarly, we have

$$T_f(r)+T_g(r)\geqslant\sum_{i=1}^{q}N^{[1]}_{Q_i(g)}(r).$$

Summing-up both sides of the above two inequalities, we have

$$\begin{aligned}2\big(T_f(r)+T_g(r)\big)&\geqslant\sum_{i=1}^{q}N^{[1]}_{Q_i(f)}(r)+\sum_{i=1}^{q}N^{[1]}_{Q_i(g)}(r)\\&\geqslant\sum_{i=1}^{q}\frac{1}{N}N^{[N]}_{Q_i(f)}(r)+\sum_{i=1}^{q}\frac{1}{N}N^{[N]}_{Q_i(g)}(r).\end{aligned}\tag{4.2}$$

From (4.2) and applying Theorem 1.1 for f and g, we have

$$\begin{aligned}2\big(T_f(r)+T_g(r)\big)&\geqslant\sum_{i=1}^{q}\frac{1}{N}N^{[N]}_{Q_i(f)}(r)+\sum_{i=1}^{q}\frac{1}{N}N^{[N]}_{Q_i(g)}(r)\\&\geqslant\frac{d}{N}\frac{q}{2N+1}\big(T_f(r)+T_g(r)\big)+o\big(T_f(r)+T_g(r)\big).\end{aligned}$$

Letting $r\to+\infty$, we get $2\geqslant\frac{d}{N}\frac{q}{2N+1}\Leftrightarrow q\leqslant\frac{2N(2N+1)}{d}$. This is a contradiction.

Hence $f=g$. The assertion a) is proved.

b) By Lemma 4.1(ii), we have $\|T_f(r)=O(T_g(r))$ and $\|T_g(r)=O(T_f(r))$. Suppose that $f\neq g$. Repeating the same argument as in a), we get the following inequality, which is similar to (4.2),

$$2\big(T_f(r)+T_g(r)\big)\geqslant\sum_{i=1}^{q}\frac{1}{N}N^{[N]}_{Q_i(f)}(r)+\sum_{i=1}^{q}\frac{1}{N}N^{[N]}_{Q_i(g)}(r).\tag{4.3}$$

From (4.3) and applying Theorem 1.1b) for f and g, we have

$$\begin{aligned}2\big(T_f(r)+T_g(r)\big)&\geqslant\sum_{i=1}^{q}\frac{1}{N}N^{[N]}_{Q_i(f)}(r)+\sum_{i=1}^{q}\frac{1}{N}N^{[N]}_{Q_i(g)}(r)\\&\geqslant\frac{d}{N}\frac{q}{N+n+1}\big(T_f(r)+T_g(r)\big)+o\big(T_f(r)+T_g(r)\big).\end{aligned}$$

Letting $r\to+\infty$, we get $2\geqslant\frac{d}{kN}\frac{q}{N+n+1}\Leftrightarrow q\leqslant\frac{2N(N+n+1)}{d}$. This is a contradiction.

Hence $f=g$. The assertion b) is proved. □

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